

一、填充題：每題 20 分，共 100 分

1. 設 $a = \sqrt{7} - \sqrt{3}$ ， $b = \sqrt{8} - \sqrt{4}$ ， $c = \sqrt{9} - \sqrt{5}$ ，則 a, b, c 大小關係為【 】

2. 已知 $x + \frac{1}{x} = 5$ ，試求下列各式之值：

(1) $x^2 + \frac{1}{x^2} =$ 【 】 (2) $x^3 + \frac{1}{x^3} =$ 【 】

3. 設 $a + b = 4$ ， $ab = 1$ ，則 $\frac{b}{a+1} + \frac{a}{b+1} =$ 【 】

4. 已知 a, b 為實數，且 $a + b = 5$ ， $a^3 + b^3 = 185$ ，求 $a^2 + b^2 =$ 【 】

5. 設 $a = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ ， $b = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ ，試求 $\frac{b^2}{a} + \frac{a^2}{b}$ 的值為【 】

一、 填充題：每題 20 分，共 100 分

1. $a > b > c$

2. (1) 23 ; (2) 110

3.3

4.33

5.52

----- << 解析 >> -----

1. 解析: $\frac{1}{a} = \frac{1}{\sqrt{7}-\sqrt{3}} = \frac{\sqrt{7}+\sqrt{3}}{4}$, $\frac{1}{b} = \frac{1}{\sqrt{8}-\sqrt{4}} = \frac{\sqrt{8}+\sqrt{4}}{4}$, $\frac{1}{c} = \frac{1}{\sqrt{9}-\sqrt{5}} = \frac{\sqrt{9}+\sqrt{5}}{4}$

顯然 $\frac{1}{a} < \frac{1}{b} < \frac{1}{c}$, 又 a, b, c 皆正數 $\therefore a > b > c$

2. 解析: (1) $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x} = \left(x + \frac{1}{x}\right)^2 - 2$
 $= 5^2 - 2 = 23$

(2) $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right)$
 $= \left(x + \frac{1}{x}\right)^3 - 3 \left(x + \frac{1}{x}\right) = 5^3 - 3 \times 5 = 110$

3. 解析: $\frac{b}{a+1} + \frac{a}{b+1} = \frac{b(b+1) + a(a+1)}{(a+1)(b+1)} = \frac{(a+b)^2 - 2ab + (a+b)}{ab + (a+b) + 1} = \frac{16 - 2 + 4}{1 + 4 + 1} = 3$

4. 解析: $a + b = 5, a^3 + b^3 = 185$

又 $a^3 + b^3 = (a+b)^3 - 3ab(a+b) \Rightarrow 185 = 5^3 - 3 \times 5 \times ab \Rightarrow ab = \frac{185 - 125}{-15} = -4$

故 $a^2 + b^2 = (a+b)^2 - 2ab = 5^2 - 2 \times (-4) = 25 + 8 = 33$

5. 解析: $a = \frac{\sqrt{3}+1}{\sqrt{3}-1}, b = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$$\therefore \begin{cases} a+b = \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{2} = \frac{8}{2} = 4 \\ ab = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}-1}{\sqrt{3}+1} = 1 \end{cases}$$

故 $\frac{b^2}{a} + \frac{a^2}{b} = \frac{a^3 + b^3}{ab} = \frac{(a+b)^3 - 3ab(a+b)}{ab} = \frac{4^3 - 3 \times 1 \times 4}{1} = 52$