

一、填充題：每題 15 分，共 105 分

1. 已知 θ 為銳角， $\sin \theta = \frac{2}{\sqrt{5}}$ ，則 $\sin 4\theta =$ 【 】
2. 求 $\sin^4 11.25^\circ + \cos^4 11.25^\circ =$ 【 】
3. 在 $\triangle ABC$ 中， $\overline{AB} = 3$ ， $\overline{BC} = 4$ ， $\angle A = 3\angle C$ ，則 $\sin^2 C =$ 【 】
4. 設 $\sin \theta = \frac{8}{5} \cos \frac{\theta}{2}$ ，則 $\cos \theta =$ 【 】
5. 設 $\tan A$ ， $\tan B$ 為 $7x^2 - 8x + 1 = 0$ 的兩根，則 $\tan \frac{A+B}{2} =$ 【 】
6. 設 $\sin \theta$ ， $\cos 2\theta$ 為 $8x^2 + 7x + a = 0$ 之兩根，則 $a =$ 【 】
7. 設 $f(x) = 8x^3 - 2x^2 - 6x + 1$ ，則以 $x - \sin 15^\circ$ 除 $f(x)$ 之餘式為 【 】

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1. $-\frac{24}{25}$

2. $\frac{6+\sqrt{2}}{8}$

3. $\frac{5}{12}$

4. -1 或 $-\frac{7}{25}$

5. $\frac{1}{2}$ 或 -2

6. $\frac{3}{4}$

7. $-\sqrt{2} + \frac{\sqrt{3}}{2}$

----- << 解析 >> -----

1. **解析** : $\sin(4\theta) = \sin(2\theta + 2\theta) = 2\sin 2\theta \cos 2\theta = 4\sin\theta \cos\theta (\cos^2\theta - \sin^2\theta)$
 $= 4 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} \cdot \left(\frac{1}{5} - \frac{4}{5}\right) = -\frac{24}{25}$

2. **解析** : $\sin^4 11.25^\circ + \cos^4 11.25^\circ$
 $= (\sin^2 11.25^\circ + \cos^2 11.25^\circ)^2 - 2\sin^2 11.25^\circ \cos^2 11.25^\circ$
 $= 1 - \frac{1}{2} (2\sin 11.25^\circ \cos 11.25^\circ)^2 = 1 - \frac{1}{2} (\sin 22.5^\circ)^2$
 $= 1 - \frac{1}{2} \cdot \frac{1 - \cos 45^\circ}{2} = 1 - \frac{1 - \frac{\sqrt{2}}{2}}{4} = \frac{6 + \sqrt{2}}{8}$

3. **解析** : 由正弦定理，可得 $\frac{3}{\sin C} = \frac{4}{\sin A} \Rightarrow 3\sin A = 4\sin C$

$\Rightarrow 3\sin(3C) = 4\sin C$

$\Rightarrow 3(3\sin C - 4\sin^3 C) = 4\sin C$

$\Rightarrow \sin C(12\sin^2 C - 5) = 0$

$\Rightarrow \sin C = 0$ (不合) 或 $\sin^2 C = \frac{5}{12}$

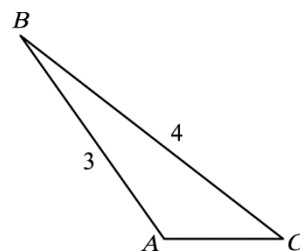
4. **解析** : $\sin\theta = \frac{8}{5}\cos\frac{\theta}{2} \Rightarrow 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \frac{8}{5}\cos\frac{\theta}{2} = 0$

$\Rightarrow 2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2} - \frac{4}{5}\right) = 0 \Rightarrow \cos\frac{\theta}{2} = 0$ 或 $\sin\frac{\theta}{2} = \frac{4}{5}$

(1) $\cos\frac{\theta}{2} = 0 \Rightarrow \cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 2(0)^2 - 1 = -1$

(2) $\sin\frac{\theta}{2} = \frac{4}{5} \Rightarrow \cos\theta = 1 - 2\sin^2\frac{\theta}{2} = 1 - 2\left(\frac{4}{5}\right)^2 = 1 - \frac{32}{25} = -\frac{7}{25}$

由(1)、(2)得 $\cos\theta = -1$ 或 $-\frac{7}{25}$





5. 解析: $\tan A, \tan B$ 為 $7x^2 - 8x + 1 = 0$ 之兩根 $\Rightarrow \tan A + \tan B = \frac{8}{7}, \tan A \tan B = \frac{1}{7}$

$$\text{所以 } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{8}{7}}{1 - \frac{1}{7}} = \frac{4}{3}$$

$$\Rightarrow \begin{cases} \sin(A+B) = \frac{4}{5} \\ \cos(A+B) = \frac{3}{5} \end{cases} \text{ 或 } \begin{cases} \sin(A+B) = -\frac{4}{5} \\ \cos(A+B) = -\frac{3}{5} \end{cases}$$

$$\Rightarrow \tan \frac{A+B}{2} = \frac{\sin(A+B)}{1 + \cos(A+B)} = \frac{\frac{4}{5}}{1 + \frac{3}{5}} \text{ 或 } \frac{-\frac{4}{5}}{1 + \left(-\frac{3}{5}\right)} \Rightarrow \frac{1}{2} \text{ 或 } -2$$

6. 解析: 兩根和 $\sin \theta + \cos 2\theta = -\frac{7}{8} \Rightarrow \sin \theta + 1 - 2\sin^2 \theta = -\frac{7}{8}$

$$\Rightarrow 16\sin^2 \theta - 8\sin \theta - 15 = 0 \text{ 得 } \sin \theta = -\frac{3}{4} \text{ 或 } \frac{5}{4} \text{ (不合)}$$

$$\Rightarrow \cos 2\theta = -\frac{7}{8} - \left(-\frac{3}{4}\right) = -\frac{1}{8}$$

$$\text{所以兩根積 } \frac{a}{8} = \left(-\frac{3}{4}\right)\left(-\frac{1}{8}\right) \Rightarrow a = \frac{3}{4}$$

7. 解析: $\because f(x)$ 除以 $x-a$ 的餘式為 $f(a)$

$$\begin{aligned} \therefore \text{所求為 } f(\sin 15^\circ) &= 8\sin^3 15^\circ - 2\sin^2 15^\circ - 6\sin 15^\circ + 1 \\ &= -2(3\sin 15^\circ - 4\sin^3 15^\circ) + (1 - 2\sin^2 15^\circ) \\ &= -2\sin(3 \times 15^\circ) + \cos(2 \times 15^\circ) \\ &= -2\sin 45^\circ + \cos 30^\circ = -2 \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \\ &= -\sqrt{2} + \frac{\sqrt{3}}{2} \end{aligned}$$